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DIMINISHING RETURNS AND LABOR MARKET ADJUSTMENTS

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We amend the canonical matching model by assuming diminishing returns to labor. We put the model to the twin test of generating a high volatility of labor market variables in response to productivity shocks (the "Shimer puzzle") and a moderate response to changes in unemployment benefits and find that it passes that test. It does not feature wage rigidity, nor is it based on a small surplus calibration. Diminishing returns introduce a distinction between *marginal* and *average* surplus. With a standard (large average surplus) calibration, we can have a small marginal surplus, and thus a strong response of hiring to productivity shocks, while obtaining a measured response of unemployment to changes in benefits.

Keywords: Diminishing Returns, Matching Frictions, Business-Cycle Fluctuations, Labor Market Policy

1. INTRODUCTION

The canonical matching model of the labor market [Pissarides (2000)] has recently been criticized for failing to generate enough volatility in labor market variables in response to a productivity shock—the so-called "Shimer puzzle." Taking labor market tightness as the key endogenous variable from which every other variable of interest can be computed, Shimer (2005) finds that the elasticity of tightness with respect to net labor productivity is one order of magnitude smaller in the standard matching model than in the data. In other words, the standard version of the matching framework does not display enough volatility in response to a productivity shock. This is deemed a puzzle because this result holds for all reasonable parameter values.[1] Costain and Reiter (2008, hereafter CR) consider a second test that the solutions to theShimer puzzle should pass. Namely, can

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a given environment deliver both a strong reaction to small cyclical changes in productivity *and* empirically plausible adjustments in unemployment to changes in labor market policies, such as unemployment benefits (UB)? We study here a frictional model where, contrary to the standard environment, firm profit is characterized by diminishing marginal revenue product of labor, and analyze the cyclical and steady-state properties of such an environment.

The two main solutions which have been proposed to solve the Shimer puzzle have weaknesses. Models where wages are rigid can generate more volatile labor markets, since wage rigidity makes profits more procyclical. They also successfully pass the CR test. However, they are subject to the Pissarides (2009) critique that empirically, wage cyclicality in new matches is high, and thus that any model should feature cyclicality in new matches.[2] The "small surplus calibration" approach of Hagedorn and Manovskii (2008, hereafter HM) does increase volatility in the labor market since a small surplus implies a stronger reaction to a given productivity shock, however, it fails the CR test. In conclusion, the solutions proposed either fail the twin—Shimer and CR—test, or pass it but are subject to the Pissarides critique.

Our framework addresses both parts of the twin test. We introduce diminishing marginal revenue product of labor by assuming that the goods market is characterized by monopolistic competition. Diminishing returns imply a distinction between the average and the (smaller) marginal surplus from a worker–firm match, which is absent from the traditional matching framework. Since the hiring decision is based on the marginal surplus, one can get a high volatility of labor market variables in response to productivity shocks. This is, however, obtained while being immune from the two main criticisms usually addressed at the solutions to the Shimer puzzle: Bargained wages are not rigid, and the average surplus (proxied by the ratio of individual match productivity z to unemployment income b) does not have to be very small, as in HM. In addition, one can show that the ratio of the elasticity of tightness with respect to productivity $\eta_{\theta z}$ to the elasticity with respect to unemployment income $\eta_{\theta b}$ is proportional to the average surplus z/b. Thus, with our large average surplus, we can generate higher volatility in response to a productivity shock than to an unemployment income shock. Hence, we solve the twin puzzle because the marginal surplus is small, but the average surplus is not.

In the standard model with constant returns to labor, the ratio of elasticities is in fact equal to z/b. However, since it does not distinguish between marginal and average surplus, it cannot at the same time feature a small surplus to solve the Shimer puzzle, and a large surplus to ensure more volatility in response to productivity than to UB changes.

In order to introduce diminishing marginal returns, we amend the standard Pissarides model by assuming monopolistic competition in the goods market. The setup is similar to Ebell and Haefke (2009) which itself is adapted from Blanchard and Giavazzi (2003). Due to imperfect competition, the marginal revenue of labor is decreasing, and thus we rely on intrafirm bargaining between the firm and its workers, rather than the standard Nash bargaining.

Quantitatively, we find a productivity elasticity of market tightness more than twice that of the standard matching model, since Shimer (2005) finds a value of 1.7, while our model generates an elasticity of 3.9—when the target elasticity is 7.6.[3] We also find that, in addition to increasing labor market volatility, the model predicts changes of unemployment in response to variations in UB in line with the data: The semielasticity of unemployment with respect to the UB replacement rate (1.6) is within the range of values estimated in the literature ([1.3, 3.1]). Finally, we can verify that contrary to the wage rigidity approach, our model is not subject to the Pissarides critique, since the wage elasticity is close to unitary, as it is in Pissarides (2009).

We conclude by checking that the ability of this model to solve the twin puzzle is not due to other mechanisms introduced by the framework, relative to the standard matching model. Indeed, monopolistic competition implies free entry of firms. Thus, a positive productivity shock brings more firms in the market, increasing the degree of competition in the economy and lessening firms' incentive to restrict output. Quantitatively, however, this is found to be a rather weak amplification mechanism.

The rest of the paper proceeds as follows. Section 2 reviews the literature. In Section 3, we describe the environment and develop the equilibrium. Section 4 calibrates and simulates the model. Finally, we conclude in Section 5.

2. LITERATURE

The main contributions at solving the Shimer puzzle can be divided into two categories. First, as alluded to above, the assumption of Nash bargaining has the direct consequence that most of any productivity increase is absorbed by wages and not by profits. Thus, Shimer (2004) proposed that assuming (exogenously) rigid wages improves the performance of the standard model, since in that case profits of firms are not adversely affected by improving labor market conditions. Rigid wages produce procyclical profit margins and thus may provide a solution to the puzzle. However, rigidity needs to be assumed in all matches—both new and old, to have a chance of approximating the volatility in the data. This approach, however, does not provide a rationale for rigidity in wages. Various other contributions have put forth ways to endogenously generate such rigidity in the context of a matching framework. Hall (2005) introduced a wage norm, whereas Hall and Milgrom (2008) broke the link between wages and aggregate market conditions by assuming that the relevant threats in negotiating were to impose a costly delay in reaching an agreement, rather than terminating the negotiations and returning to unemployment. Intuitively, it is equivalent to putting less weight on aggregate market conditions in determining wage.

In parallel to these contributions, however, Pissarides (2009) stressed that theoretically, rigidity in existing (old) matches does not matter.[4] However, he found empirically that wage cyclicality in new matches is high, and thus that any model should feature cyclicality in new matches. Mortensen and Nagypál (2007) and

Hornstein et al. (2005, 2007) also stressed the point that for wage rigidity to lead to enough volatility, it must also be the case that labor shares are very large, too large empirically.

HM showed that the standard matching model can deliver enough volatility if two conditions are met: Profits are both strongly procyclical *and* small—the "small surplus approach." After a thorough calibration, they indeed replicated empirical elasticities.[5] However, their approach has been criticized on the grounds that some of their parameters may seem unrealistic. Instantaneous utility derived from unemployment (inclusive of home production and value of leisure) is 95% of output produced on the job—resulting in a surplus flow when employed of less than 3% over unemployment [Mortensen and Nagypál (2007)].[6]

The other test we impose on the model is its ability to match the documented effects of policy on unemployment. This is inspired by CR, who look at the Shimer Puzzle from a different perspective. They find that there is no set of parameters for which the basic matching model could replicate *both* the cyclical behavior of labor market variables *and* the effect of labor market policies on the stationary unemployment rate—although calibrations do exist which achieve these goals separately. For example, a parameterization able to replicate the cyclical properties of the labor market would predict much too strong a reaction of the unemployment rate to changes in UB. It is clearly a criticism of the small surplus approach of HM, since the solution these authors suggest brings other problems of its own. Among all the fixes CR consider, sticky wages improves the basic model's cyclical performance without delivering counterfactual implications on the policy dimension. However, they model wage stickiness in an ad hoc fashion, by assuming that the worker's bargaining power is negatively correlated with technology, and that approach, of course, is subject to the Pissarides critique.

Finally, our environment is based on a matching model of the labor market, coupled with monopolistic competition in the goods market. To our knowledge, this particular combination of setups was initiated by Blanchard and Giavazzi (2003). Ebell and Haefke (2009, hereafter EH) use that setup to analyze product market deregulation in the United States in the 1980s.[7] We follow EH's framework but study a much different question.[8]

3. MODEL

The economy is comprised of a monopolistically competitive goods market and a frictional labor market. The assumption of monopolistic competition is taken as a natural way to model diminishing marginal returns to labor.

3.1. The Goods Market

A unit mass of identical households consume a composite good aggregated from a continuum of intermediate goods indexed by j, each produced by a single firm.

Their preferences are given by

$$\left(\int_0^n c_j^{\frac{\sigma-1}{\sigma}}\,dj\right)^{\frac{\sigma}{\sigma-1}},$$

where σ is the elasticity of substitution across the n varieties. Utility maximization by households leads to a demand for good j given by $y_j = (p_j/P)^{-\sigma}Y$, where p_j is the price of good j, P the price of the composite good, and Y is aggregate output. For simplicity, we omit from now on the sectorial index j and also normalize the price index to unity. The price p can thus be interpreted as the relative price of that intermediate variety. Aggregate demand for an intermediate good is thus

$$y = p^{-\sigma}Y. \tag{1}$$

We assume, as in Blanchard and Giavazzi (2003) and EH, that the elasticity of demand for intermediate goods σ depends on the measure of firms n. In particular, the greater the measure of firms, the more elastic the demand.

3.2. The Labor Market

The labor market can be described as follows. Time is continuous. There is a mass one of workers who can either search for a job or be employed. There is an endogenously determined measure of active firms [of mass $n(t)$]. The process by which firms and workers match is frictional. In particular, firms have to pay a cost κ in terms of the final good (whose price has been normalized) for each vacancy they post. This enables them to meet workers, at a rate proportional to the measure of vacancies they post. In particular, a firm that posted v vacancies meets a worker at rate $vp_f[\theta(t)]$, whereas a worker meets a firm at rate $p_w[\theta(t)]$. Meeting rates only depend on market tightness $\theta(t)$, defined as the ratio of the measure of vacancies posted by all the firms to the measure of unemployed workers $u(t)$, as implied by the assumptions of random matching and constant returns to scale in the matching technology. Flows out of employment are due to two types of breakdowns: An individual job within a firm may break down at rate δ_s, and the entire firm may exit the market at rate δ_e. We denote the rate at which a worker may lose his job, regardless of reason, as $\delta = \delta_e + \delta_s$.

To become active, a firm must pay a one-time entry cost c_e in terms of the final good, comprising regulatory entry costs and technological costs. Once in the market, productivity z is the same for all firms. A firm with l employees produces a flow of zl units of its good, sold at a price $p(l,t)$. The firm pays each of its employees a flow wage $w(l,t)$. Notice that, because of the diminishing marginal revenue product of labor, the negotiated wage is a function of l. Workers without a job receive a flow of income while unemployed equal to b, in terms of the final good.

Time is discounted at rate r. Let $M^f(l,t)$ denote the value of a firm matched with l employees at date t. The Hamilton–Jacobi–Bellman equation is given by

$$(r+\delta_e)M^f(l,t) - M_t^f(l,t) = p(l,t)zl - w(l,t)l - \delta_s l M_l^f(l,t) + \max_v\{-\kappa v + p_f[\theta(t)]vM_l^f(l,t)\}, \quad (2)$$

where the subindex $i \in \{l,t\}$ on the value function represents its derivative with respect to the variable i. Lifetime firm value is discounted both for the passage of time and the possibility of exit. A firm makes a flow profit from selling its good and paying its workforce. It must also choose vacancy posting optimally, and may lose a measure $\delta_s l$ of workers per unit of time.

Let $M^w(l,t)$ be the value to a worker of being employed at a firm with l employees, and $S^w(t)$ the value of being unemployed, both at date t. The former satisfies

$$rM^w(l,t) - M_t^w(l,t) = w(l,t) + \{p_f[\theta(t)]v(l,t) - \delta_s l\}M_l^w(l,t) + \delta[S^w(t) - M^w(l,t)], \quad (3)$$

where $v(l,t)$ denotes the optimal vacancy choice. The value to a worker from being employed is derived from receiving a flow wage, but also accounts for the possibility of losing employment, either from job or firm breakdown. Finally, the value of the match may change because the firm size changes, due to vacancy posting or attrition. The value of unemployment satisfies

$$rS^w(t) - S_t^w(t) = b + p_w[\theta(t)]\mathbf{E}[M^w(l,t) - S^w(t)], \quad (4)$$

where the expectation sign is taken with respect to the firm size in future matches. Its value is derived from receiving a flow of income while searching, and is augmented by the capital value from finding a job.

We follow Cahuc et al. (2008), EH, Smith (1999), and Stole and Zwiebel (1996) and assume that negotiations proceed as if the firm were negotiating with the marginal worker. The bargaining solution thus determines marginal firm surplus and worker surplus, i.e.,

$$\beta M_l^f(l,t) = (1-\beta)[M^w(l,t) - S^w(t)], \quad (5)$$

according to the worker bargaining power β.

Finally, firms are free to enter the market (with zero employee), thus they exploit all profit opportunities and

$$M^f(0,t) = c_e, \quad \text{for all } t. \quad (6)$$

As firms enter the goods market, the elasticity of demand increases and the monopoly power of firms decreases. We follow Blanchard and Giavazzi (2003) and EH and assume that the elasticity is proportional to the number of firms, i.e.,

$$\sigma(t) = \widehat{\sigma} n(t). \quad (7)$$

The unemployment rate $u(t)$ is the result of flows out of employment at rate δ, reduced by flows into employment at the rate $p_w[\theta(t)]$. Thus,

$$u_t(t) = \delta[1 - u(t)] - p_w[\theta(t)]u(t). \quad (8)$$

We can now define an equilibrium:

DEFINITION 1. *An equilibrium is comprised of (i) value functions* $S^w(t)$, $M^w(l,t)$, *and* $M^f(l,t)$, *(ii) prices* $w(l,t)$ *and* $p(l,t)$, *(iii) a number of firms* $n(t)$ *and a firm size* $\widehat{l}(t)$, *and (iv) a labor market tightness* $\theta(t)$ *and an elasticity of demand* $\sigma(t)$, *such that the value functions satisfy the Hamilton–Jacobi–Bellman equations (2)–(4); the bargaining equation (5) holds and the relative price* $p[\widehat{l}(t),t] = 1$ *under symmetry; there is free entry of firms, i.e., (6) is satisfied, and vacancy posting is optimal, i.e., (9) is satisfied; and the unemployment rate satisfies* $1 - u(t) = n(t)\widehat{l}(t)$ *and* $\sigma(t)$ *is given by (7).*

Notice that, in order to simplify the notation, we anticipated in the above definition that equilibrium is characterized by a unique firm size. We verify it in the next section. To the usual value function/prices/allocations formulation of equilibrium, we added labor market tightness and degree of competition. Each reflects "tightness" in its respective market. We will see that they are jointly determined in equilibrium.

3.3. Characterization of Equilibrium

In the standard matching model à la Pissarides (2000), the endogenous variables of interest are market tightness, wages, and unemployment. Among these, only unemployment does not adjust right away to its steady-state value [this can be checked by inspection of (8)].[9] In our model, we add firm size as another endogenous variable. However, we show below that firm size does not exhibit any persistence, and that in our environment too, the only variable with sticky dynamics is unemployment.[10]

To understand how firm size adjusts, consider the first-order condition with respect to vacancy posting, which equalizes marginal benefit with marginal cost of a vacancy. From (2), we have

$$\kappa = p_f[\theta(t)]M_l^f(l,t). \quad (9)$$

This implies that optimal firm size is independent of current size, so that in equilibrium all firms have the same size. That is, entrants post a very large measure of vacancies for a very brief time to reach that size, while incumbents post just enough vacancies to remain at that optimal level.

We know from Mortensen and Nagypál (2007) and Shimer (2005) that steady-state elasticities are very good approximations for elasticities generated by an equivalent fully stochastic model with aggregate shocks.[11] Thus, in the rest of the paper, we analyze the steady-state version of equilibrium by dropping all time indices and denoting steady-state variables by a "*".[12]

We start by deriving the wage function and show in the appendix that the wage rule equates the firm's marginal income flow from the worker with the worker's surplus flow,[13] up to their respective bargaining power, i.e.,

$$\beta \frac{d}{dl}[p(l)zl - w(l)l] = (1-\beta)[w(l) - rS^w].$$

This is a differential equation in $w(l)$ to solve, which can be rewritten as

$$w(l) + \beta w_l(l)l = \beta \frac{d}{dl}[p(l)zl] + (1-\beta)rS^w.$$

From the demand for the intermediate variety, equation (1), we know that the derivative on the right-hand side is equal to $\frac{\sigma-1}{\sigma}zp(l)$. This equation belongs to a class of problems for which a generic solution is known. This equation is solved in Delacroix (2006), who finds that[14]

$$w(l) = rS^w + \beta\left[\frac{\sigma-1}{\sigma-\beta}zp(l) - rS^w\right]. \quad \textbf{(10)}$$

We now look for the steady-state equilibrium allocations and prices. Combining (9) with (2) differentiated with respect to l, we obtain (taking into account the first-order condition) that

$$\frac{d}{dl}[p(l)zl - w(l)l] = (r+\delta)\kappa/p_f(\theta).$$

Using (1) and (10) to compute the left-hand side, and expressing the result at steady-state values, it yields

$$p^* = \frac{1}{z}\frac{\sigma^*-\beta}{\sigma^*-1}[w^* + (r+\delta)\kappa/p_f(\theta^*)]. \quad \textbf{(11)}$$

This relationship is similar to a markup condition, since it takes into account the effect of the firm's employment decision on both relative price and wage.

Combine (3)–(5) and (9) to get[15]

$$w^* = b + \frac{\beta}{1-\beta}[r+\delta+p_w(\theta^*)]\kappa/p_f(\theta^*). \quad \textbf{(12)}$$

As expected, the wage depends positively on unemployment income and the worker bargaining power, and is an increasing function of market tightness.

Because of symmetry inherent in the problem, the relative price

$$p^* = 1. \tag{13}$$

Entrants post a measure v_e^* of vacancies to reach optimal firms size, whereas incumbents post a measure v_i^* to maintain it, thus $p_f(\theta^*)v_e^* = l^*$ and $p_f(\theta^*)v_i^* = \delta_s l^*$. Because of the immediate adjustment of entrants to the optimal firm size, it follows that $M^f(0) = -\kappa v_e^* + M^f(l^*)$. Using (2) to compute $M^f(l^*)$, and inserting it into (6), we obtain the following free entry condition:

$$\frac{p^* z - w^* - \kappa\delta_s/p_f(\theta^*)}{r + \delta_e} l^* = c_e + \kappa l^*/p_f(\theta^*). \tag{14}$$

Lifetime profits discounted for the passage of time and the impermanence of incumbent firms cover the initial entry and vacancy posting costs.

Employment is equal both to the number of firms times average firm size, and to the size of the labor force times its employment rate. Thus, we have

$$n^* l^* = 1 - u^* = \frac{p_w(\theta^*)}{\delta + p_w(\theta^*)}, \tag{15}$$

where the employment rate is obtained by equating flows in and out of a job.

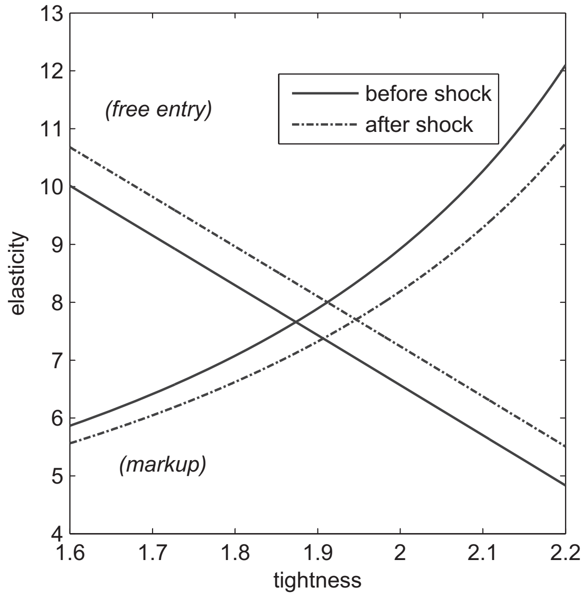
Finally, equation (7) expressed at steady state as

$$\sigma^* = \widehat{\sigma} n^* \tag{16}$$

implies a positive link between entry and the degree of competition.[16]

In conclusion, a steady-state equilibrium is characterized by a wage w^*, a relative price p^*, a market tightness θ^*, a measure of firms n^*, a firm size l^*, and an elasticity of demand σ^* satisfying (11)–(16).

Intuitively, one can graphically picture equilibrium in (θ, σ)-space as the intersection of two curves simultaneously determining tightness θ^* in the labor market and the degree of competition σ^* in the goods market. [Alternatively, because of (16), we could graph these relationships in (θ, n)-space.] The first curve is obtained by combining equations (11)–(13), and is thus derived from firms' optimizing behavior and their strategically taking into account the relationship between employment decision, wage paid, and the price of the produced good. Since wage and recruiting costs depend on market tightness, this equivalent of a "markup condition" implies a first relationship between θ and σ. It has a positive slope, because when the firm-level elasticity of demand σ is higher, the markup is lower. With wage and recruiting costs increasing in tightness, this implies a positive relation between θ and σ. The second relation combines equations (14)–(16) and is akin to a "free entry of firms condition." It exhibits a negative slope, because when the firm-level elasticity of demand is higher (and thus more firms are present in the market), the value of entry diminishes. The free entry condition is satisfied when costs, and thus tightness, are lower.[17] When a positive aggregate productivity shock hits the economy, both curves move towards higher tightness. We quantitatively evaluate the equilibrium effect of such a shock in the next section.



4. NUMERICAL WORK

4.1. Calibration

Since our equilibrium features both a goods and a labor market, the calibration targets characteristics of each market. In particular, we choose parameters to reproduce labor market dynamics by matching the unemployment and job finding rates for workers, as well as firms' characteristics such as average firm size and firm survival rates. The parameterization also ensures a degree of competition in the goods market consistent with average markups in US manufacturing.

The model is calibrated to the United States for the period 1980–2000, with a month as the time period. The rate of time preference r is set to match an annual steady-state interest rate of 4.8%. The exit rate for firms δ_e replicates a one-year survival probability equal to 78.95% as estimated from the Business Dynamics Statistics data (79.8% with BLS data). The job breakdown rate δ_s ensures a steady-state unemployment rate of 6.5%, when combined with a monthly job finding rate of 0.45 as estimated by Shimer (2005).

Productivity z is normalized to 1. The term b includes not only UB, but also the value of leisure and home production. The literature has shown it to be an important parameter, since a high value of b as in the small surplus approach can generate really high values of elasticities. We conservatively set $b = 0.6$ to account for an average replacement rate of 30% in the United States and an equivalent value for leisure and home output. This is lower than the value of 0.71 estimated in Hall and Milgrom (2008). The period vacancy posting cost is set at $\kappa = 0.6$, which is in the mid-range of values estimated in the literature.[18] We use the Den

Haan et al. (2000) matching function, i.e.,

$$M(U, V) = \frac{UV}{(U^{\lambda} + V^{\lambda})^{1/\lambda}},\ \lambda > 0,$$

which ensures that matching probabilities always lie in [0, 1] while retaining the usual properties of monotonicity, concavity, and constant returns to scale. Den Haan et al. report a monthly job filling rate of 0.24. With the job finding rate from Shimer (2005), the resulting market tightness implies a "matching" parameter $\lambda = 0.639$.

We target a markup over marginal cost $\mu^* = \sigma^*/(\sigma^* - 1)$ equal to 1.15 as reported in Martins et al. (1996), who find an average markup of 15% in US manufacturing over the 1980–1992 period, with most industries in the (5%–25%) range. This implies a firm-level elasticity of demand equal to $\sigma^* = 7.7$. Combining (15) and (16), and with an average firm size $l^* = 20$ employees as reported in Axtell (2001) and by the Census Bureau, we obtain $\widehat{\sigma} = 164$. Notice that, because we choose to match the observed unemployment rate, markup, and average firm size, the product of $\widehat{\sigma}$ and the size of the labor force (here normalized to 1) is thus set. Increasing the labor force would reduce $\widehat{\sigma}$ and increase the number of firms proportionally.

There remain two important parameters to calibrate, β and c_e. As mentioned in Section 2, a low worker bargaining power helps in generating a high elasticity $\eta_{\theta,z}$. Very often, the literature assumes that the bargaining power satisfies the Hosios (1990) rule for efficiency. There is no a priori reason that the equilibrium be efficient. In addition, the usual Hosios rule is not sufficient to ensure efficiency in our setup, because of the distortions brought by monopolistic competition. HM choose the bargaining power to match an estimated elasticity of real wages to productivity. They estimate a relatively low elasticity of aggregate wages—making profits more procyclical, but are not immune to the Pissarides (2009) critique that the relevant wage elasticity is the one for workers in new jobs, hence should be higher than estimated by them. We choose the bargaining power to ensure that equations (11)–(13) are satisfied with the targets mentioned above and the parameters already calibrated, and obtain $\beta = 0.14$. Although larger than the value in HM, this is relatively low compared to what is used in the literature. It is to be noticed, however, that Delacroix (2006), who uses a similar setup with unionized and nonunionized sectors, needs a bargaining power of 0.045 to match the union wage premium. Also, estimated rent splitting parameters are all below 0.2 [Christofides and Oswald (1992), Abowd and Lemieux (1993), Blanchflower et al. (1996), Van Reenen (1996), Hildreth and Oswald (1997)].

Finally, we use (14) to determine the one-time entry cost $c_e = 97$. This includes all entry costs, both policy and technological, and represents 5 months of output per worker hired. That such costs are much larger than the policy costs estimated in Djankov et al. (2002) is consistent with what Felbermayr and Prat (2011) find with a similar framework.

Table 1 summarizes the calibration.

TABLE 1. Calibrated parameters (US 1980–2000)

Parameter		Value	Source/Target
z	Labor productivity	1	Normalization
r	Discount rate	0.0039	Annual real interest rate = 4.8%
b	Unemployed utility	0.6	Replacement rate + home production
κ	Vacancy cost	0.6	Literature
$\widehat{\sigma}$	Elasticity parameter	164	Markup, average firm size, equations (15) and (16)
δ	Match breakdown rate	0.0313	Unemployment rate = 6.5% and job finding rate = 0.45 (Shimer)
δ_e	Firm exit rate	0.0197	1-year firm survival rate = 79% (BDS)
δ_s	Job separation rate	0.0116	Derived from δ and δ_e
λ	Matching parameter	0.639	Job filling rate = 0.24 (den Haan et al.)
β	Workers' bargaining power	0.14	Markup equation (11)
c_e	Entry cost	97	Entry equation (14), steady-state equation (15)

4.2. Simulations

In this section, we simulate the environment to check whether a model with diminishing returns to labor can generate higher labor market volatility than the standard matching model. We also put our model to the twin test of verifying whether it is able to improve on the cyclical properties of the basic matching model, while predicting variations in the average unemployment rate in reaction to changes in UB in line with the data. As the model is successful in both dimensions, we highlight why diminishing returns allowed for that result. We also verify that the model is not subject to the Pissarides critique and check that no other aspect of the model differing from the standard matching environment contributes to that result. Finally, we undertake sensitivity analysis relative to parameters known to have a strong effect on volatility in the standard environment.

In order to see how well the model performs in terms of labor market volatility, we compare it to the data and the standard matching model. In particular, we report steady-state elasticities of labor market tightness with respect to productivity $\eta_{\theta,z}$, after a 1% increase[19] in productivity z.

Shimer (2005) reports targets of 19.1 for the relative volatility of labor market tightness (9.5 for unemployment and 5.9 for the job finding rate). These targets have been estimated by taking the ratio of the standard deviations of tightness and productivity. However, these should not be the target for our model which features only one type of shock. As pointed out in Mortensen and Nagypál (2007) and Pissarides (2009), sources of fluctuations other than productivity shocks generate the observed volatility of θ, u, and p_w. To provide an empirical target suited for a model with only one type of perturbation, the relative standard deviations must

TABLE 2. US data vs. Shimer (2005) vs. benchmark

	Variable x	θ	u	p_w
US data				
Elasticity†	$\eta_{x,z}$	7.56	−3.88	2.34
Semielasticity $\overline{\eta}_{u,b}$	[1.3; 3.1]			
Shimer (2005)				
Elasticity	$\eta_{x,z}$	1.75	−0.43	0.50
Semielasticity $\overline{\eta}_{u,b}$	0.45			
Benchmark with diminishing returns				
Elasticity	$\eta_{x,z}$	3.89	−1.40	1.52
Semielasticity $\overline{\eta}_{u,b}$	1.57			

†Adjusted from Shimer (2005) for compatibility with a model with a single source of fluctuations.

be multiplied by the coefficient of correlation with productivity $\rho_{\theta,z} = 0.396$, $\rho_{u,z} = -0.408$, and $\rho_{p_w,z} = 0.396$ (to get what would be the coefficient obtained from regressing these variables on productivity). The adjusted targets are thus 7.56 for market tightness, −3.88 for unemployment, and 2.34 for the job finding rate. Despite this, the Shimer puzzle remains to be solved, but the undertaking is not quite as daunting.

Table 2 shows that the model delivers an elasticity of 3.89 for market tightness, −1.40 for unemployment, and 1.52 for the job finding rate. This is slightly above 50% of the target elasticity of market tightness, but much higher than the one reported in Shimer (2005). It also improves the performance of the standard matching model for the other elasticities.

For the second part of the test, we compute $\overline{\eta}_{u,b}$, the semielasticity of unemployment with respect to the UB replacement rate implied by the model $[d\ln u/d(b/z)]$. Layard and Nickell (1999) report a semielasticity of 1.3, while CR, using a broad set of methodologies over a longer time period, find estimates around 2, but ranging from 1.3 to 3.1. Simulating our model,[20] we obtain a steady-state semielasticity of the unemployment rate with respect to changes in the b/z ratio equal to 1.57. In other terms, our mechanism increases volatility of labor market variables in response to productivity shocks, while still delivering responses in line with data when policy changes are considered.

Thus, our environment improves on the basic matching model's cyclical properties *and* is able to match the response of the unemployment rate to UB. This is because with diminishing marginal revenue product, there is a difference between the firm marginal and average surplus. The calibrated parameters generate both a large average firm surplus—proxied by the ratio of productivity to unemployment income z/b, and a small marginal firm surplus. Thus, because hiring decisions depend on the firm marginal surplus, we have a stronger reaction to productivity shocks than the standard model.

However, one may wonder why unemployment responds moderately to changes in benefits. Again using equations (11)–(13) and with a fixed degree of competition for analytic simplicity, we obtain a relation between market tightness on the one hand, and z and b on the other hand, which can be used to find that

$$\left|\frac{\eta_{\theta z}}{\eta_{\theta b}}\right| = \frac{\sigma - 1}{\sigma - \beta}\frac{z}{b}.$$

In effect, the ratio of the elasticities of market tightness with respect to z and to b, respectively, is proportional to the average surplus. Since we do not need to rely on a calibration à la HM (i.e., z/b close to 1) to generate a strong response to productivity shocks, our model can generate a stronger response to productivity shocks than to changes in UB. We are thus immune to the criticism of the HM calibration,[21] since our replacement rate $b/z = 0.6$.

Notice (by taking the degree of competition arbitrarily high) that in the standard matching model, the ratio of elasticities is equal to z/b. Such an environment could generate a more volatile response to productivity shocks than to UB changes with a high enough ratio of productivity to benefits. However, exactly because the surplus would be large, it would fail to generate enough volatility to productivity shocks, as established in HM. When average and marginal surplus are the same, it is difficult to solve the twin puzzle.

We can also check that, contrary to the wage rigidity approach, our model is not subject to the Pissarides critique. With Nash bargaining as in the traditional framework, worker surplus is equated to the firm average surplus, up to a bargaining parameter. It is straightforward to show that the Nash wage depends both on productivity and on labor market conditions [through S^w and thus θ, since $w^{\text{Nash}} - rS^w = \beta(z - rS^w)$]. With intrafirm bargaining on the other hand, the wage is obtained by equating worker surplus and firm marginal surplus [equation (5)], and with optimal vacancy posting implying that the latter is equal to expected vacancy costs [equation (9)], we can conclude that the wage is only a function of market tightness θ and not of productivity z. Hence, if the wage reacts to changes in productivity, it can only be through a general equilibrium effect on market tightness.

Quantitatively, we find that the wage displays a high degree of flexibility, since its wage elasticity $\eta_{w,z} = 0.93$. This implies that the general equilibrium effect is strongly at play. Pissarides (2009) finds that the standard model also features a wage elasticity close to unity. We thus do not rely on a low-wage elasticity to obtain our result and are immune to the Pissarides critique. This is corroborated by the fact that the elasticity of the profit rate is low (equal to 0.09). Another check of the model is to compare its wage elasticity with the one estimated in Pissarides, who finds that the elasticity of wages for job changers (the one relevant for models where bargaining resolves the division of match surplus at the beginning of the relationship) ranges between 0.9 and 1.7.

We can also verify that the other mechanism introduced by our model relative to the standard matching model does not contribute to the result. In order to quantify

TABLE 3. Robustness analysis

b	$\eta_{\theta,z}$	$\overline{\eta}_{u,b}$	β	$\eta_{\theta,z}$	$\overline{\eta}_{u,b}$
0.4	2.11	0.84	0.2	3.67	1.47
0.5	2.73	1.09	0.4	3.25	1.29
0.6	3.89	1.57	0.6	2.98	1.16
0.7	6.83	2.84	0.8	2.76	1.06

Notes: When varying b, (β,c_e) are recalibrated. When varying β, (κ,c_e) are recalibrated.

the contribution of entry to labor market volatility, we simulate the effect of a 1% increase in productivity on equilibrium tightness, and also compute tightness after the same shock when the number of firms is fixed at its initial equilibrium value, hence turning off the amplification mechanism provided by entry. The difference in tightness after the shock under the two scenarios can thus be used to assess the contribution of entry to volatility. We find that with fixed entry, the elasticity of market tightness is 3.62. When compared to an equilibrium elasticity of 3.89, this implies that entry does contribute to increasing labor market volatility, but nonetheless provides weak amplification. We tried a number of alternative calibrations and, in each case, reached the same conclusion.

As a robustness exercise, we can check that the parameters known to be key in the small surplus approach, namely unemployment income b and bargaining power β, play the same role in our environment. Indeed, a higher (lower) than calibrated b (β) can drastically increase volatility after a productivity shock, as shown in Table 3: With a higher unemployment income, relative variations in z induce greater variations in "net productivity" $z - b$, and a lower β implies more rigid wages, and thus more procyclical profits. Our calibration produced more conservative parameters than the small surplus approach, and thus is not subject to the criticism often applied to it. For similar reasons, the semielasticity $\overline{\eta}_{u,b}$ is increasing in the calibrated b and decreases with β.

5. CONCLUSION

We considered a frictional labor market, where labor exhibited diminishing marginal returns, and put it to the twin test of solving the Shimer puzzle *and* matching the response of unemployment to changes in UB. The most commonly suggested solutions to the Shimer puzzle either fail the second part of the test or are subject to the Pissarides critique, so that the literature has not successfully passed that test yet.

Quantitatively, we find an elasticity of market tightness more than twice that of the standard matching model. At the same time, the model also predicts changes in unemployment in response to variations in UB in line with estimated values. With decreasing marginal revenue product of labor, it is possible to have a small marginal surplus together with a larger average surplus. The former implies that hiring is sensitive to productivity, while the latter implies that unemployment is not responding as strongly to UB as it is to productivity shocks. A model without diminishing marginal returns could not achieve this. Our approach thus passes the twin test, while being immune from the "small surplus calibration" and the Pissarides critiques.

NOTES

1. The reason why the standard model fails is that under the commonly assumed Nash split of match surplus, wages depend both on match productivity *and* on aggregate labor market conditions. Firms react to a positive shock to productivity by posting more vacancies, thus increasing labor market tightness and enhancing workers' bargaining position. In other terms, a shock has the combined effect of increasing the size of match output *and* of changing its split between worker and firm. Most of the productivity increase is absorbed by wages, in effect leaving firms with profit opportunities which are not changed enough to replicate empirical elasticities.

2. In addition, the literature shows that not all forms of wage rigidity can solve the puzzle.

3. This is lower than what is given in Shimer (2005) but is corrected for the fact that, in the data, sources of fluctuations other than productivity shocks generate the observed volatility of θ.

4. Nash bargaining determines the split of the joint lifetime surplus. Hence, present value of wages and profits is what is settled by bargaining. As a result, assuming Nash bargaining for newly formed matches, but some fixed wage once the relationship is in place, would lead to the same implications for lifetime profits and thus vacancy posting as assuming Nash bargaining throughout the duration of the match.

5. We will see later that by matching the elasticity of market tightness to productivity reported in Shimer (2005), they, in fact, overshot what should have been their target.

6. The literature has considered two other lines of research. Shimer (2005) suggests private information as a candidate solution. However, Bruegemann and Moscarini (2010) showed that for a wide class of asymmetric information models, worker rents generated by private information are not countercyclical enough to induce high volatility in labor market variables. Second, cost structures can be added, which increase labor market elasticities. Mortensen and Nagypál (2007) include a fixed turnover cost, independent of match productivity. As a result, the opportunity cost from match continuation is higher and net payoffs more sensitive without having to assume high unemployment utility, making the model more volatile. Pissarides (2009) adds a fixed matching cost. Although the mechanisms highlighted in these two papers increase labor market volatility, the costs introduced are difficult to measure and the authors do not attempt to do so.

7. Felbermayr and Prat (2011) also study product market regulation in an environment with monopolistic competition and matching frictions, by differentiating between different types of regulations. Cacciatore and Fiori (2016) study the cyclical and stationary implications of product and labor market deregulation. Similar frameworks combining Melitz (2003) industry structure with matching frictions have been used in models of trade [Helpman and Itskhoki (2010), Helpman et al. (2010), Felbermayr et al. (2011)].

8. Elsby and Michaels (2013) feature a matching model with diminishing returns to labor, by exogenously assuming diminishing marginal returns in the production technology. They thus also feature a distinction between marginal and average surplus. They rely on that distinction to build and calibrate a model which delivers both reasonable labor market volatility and the right magnitude in worker turnover. The objective of their paper is different from ours since they do not address the CR test.

Acemoglu and Hawkins (2014) also have a matching model with diminishing returns to labor. Their focus is on the persistence properties of such an environment.

9. Since all profit opportunities are immediately exploited at all times, vacancies are a jump variable. As a consequence, wages adjust immediately as well.

10. Acemoglu and Hawkins (2014) derive the dynamics of wage adjustment in an environment similar to ours (including the nature of bargaining), but with a built-in persistence in firm size due to convex vacancy costs. This is exactly the gradual adjustment in firm size which creates a slow change in wages. We show below that size adjustment is immediate under our setup, implying the same for wages, which only depend on market tightness.

11. This is the case if the productivity process is persistent enough—either because shocks occur infrequently or because productivity changes conditional on a shock are small. The latter is the case in the Shimer simulation.

12. This also implies that the expectation term in (4) can be dropped out.

13. When bargaining, vacancy costs are sunk.

14. In the basic matching model, we would obtain $w = rS^w + \beta(z - rS^w)$, the wage being equal to the worker's outside option plus a share β of the (flow) surplus. With our framework, the interpretation is the same, except that we use the *marginal* flow surplus.

15. Notice that we use the fact that the expectation term drops in (4) because optimal firm size is independent of current size. Also, since, for incumbent firms, vacancies just replace jobs lost to attrition (δ_s-breakdowns), the term $p_f v - \delta_s l = 0$ in (3).

16. In a previous version of the paper, we developed an environment where the link between entry and competition was not assumed, but endogenously determined. In particular, the environment was characterized by multiple sectors, with Cournot competition and free entry within each industry. The elasticity was found to be linear in the number of firms per industry. Using that environment gave very similar results.

17. It is positively sloped for *very* low values of θ, theoretically resulting in two equilibria, one with low market tightness and high market power, and another one with higher tightness and lower market power. The first one, however, is empirically irrelevant since its unemployment is over 90% and its markup around 70%. Thus, for the numerical work, we focus on the only empirically sensible equilibrium.

18. See Elsby and Michaels (2013), Hagedorn and Manovskii (2008), Hall (2005), Hornstein et al. (2005, 2007), Miyamoto (2011), Petrosky-Nadeau (2014), Pissarides (2009), and Shimer (2005).

19. As mentioned in Acemoglu and Hawkins (2014), an increase in optimal firm size takes place through increased hiring, while a decrease in target size takes place both through regular attrition (δ_s-shocks) and reduced hiring. Only for very large shocks would firing be necessary.

20. Notice that, as alluded to in Section (4.1), the term b includes more than just UB, with only half of its value coming from such benefits. Computing the semielasticity taking this into account has a very small numerical impact on our simulated results.

21. Rogerson et al. (2009) come to the defense of HM, which is criticized by CR for having a calibration that fails the twin test. They argue that elasticities relevant for small changes do not necessarily apply to large ones in the presence of fixed factors. Even though the point is well taken, the model developed and the accompanying simulations are just illustrative. Our point is that one does not need a small average surplus to solve the Shimer puzzle.

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APPENDIX: DERIVATION OF THE WAGE FUNCTION

Rewrite (3) to find an expression for the worker surplus,

$$(r+\delta)[M^w(l)-S^w] = w(l) - rS^w + [p_f(\theta)v(l)-\delta_s l]M_l^w(l).$$

Differentiate (5) with respect to l to obtain

$$\beta M_{ll}^f(l) = (1-\beta)M_l^w(l).$$

Inserting (5) and the equation just obtained into the equation at the top of the page, we get

$$(r+\delta)\beta M_l^f(l) = (1-\beta)[w(l)-rS^w] + [p_f(\theta)v(l)-\delta_s l]\beta M_{ll}^f(l).$$

Differentiating (2) with respect to l gives

$$(r+\delta)M_l^f(l) = \frac{d}{dl}[p(l)zl - w(l)l] + [p_f(\theta)v(l)-\delta_s l]M_{ll}^f(l),$$

using the first-order condition with respect to vacancies.

Finally, comparing the last two equations yields

$$\beta\frac{d}{dl}[p(l)zl - w(l)l] = (1-\beta)[w(l)-rS^w].$$